Homework Cover Page Format

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Date of submission: 3/3/2026

Problem set number: #4

Due date: 3/3/2026

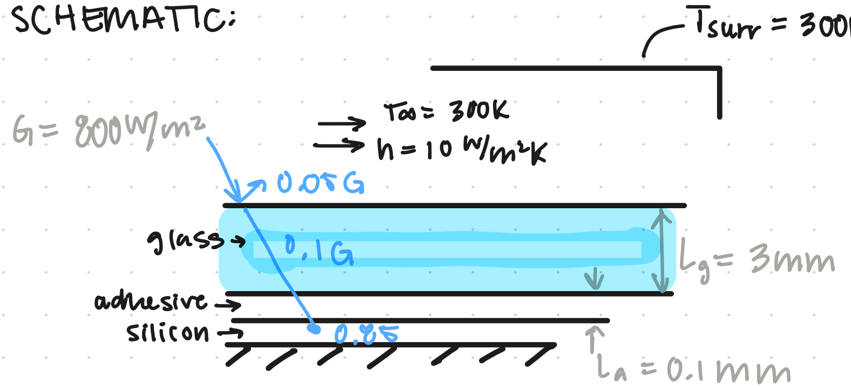
Acknowledgement: worked with TA's in Monday & Tuesday office hours

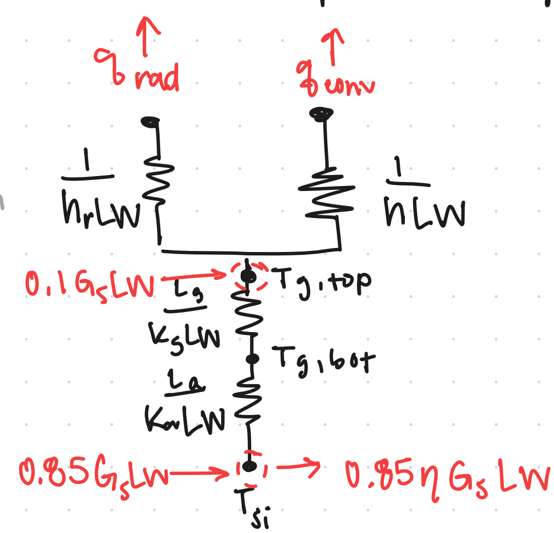
1 KNOWN: $T_{surr} = 300K$ $T_\infty = 300K$ $h = 10\ W/m^2K$ $G = 800\ W/m^2$
$k_g = 1.5\ W/mK$ $k_a = 1\ W/mK$ $\eta = a - bT_{Si}$ $a = 0.553$ $b = .001\ K^{-1}$
$R_c = 10^{-4}\ \frac{m^2K}{W}$ $G = 800\ W/m^2$ $h = 10\ W/m^2K$ $h_{rad} = 5\ W/m^2K$

FIND: A. Thermal resistance network + the thermal resistance of each
B. calculate top glass layer $T_{g,top}$ + T_s
C. calculate η + power

ASSUMPTIONS: no contact resis. btwn glass + adhesive, SS, 1D, constant prop, negligible thermal diff w/ silicon layer

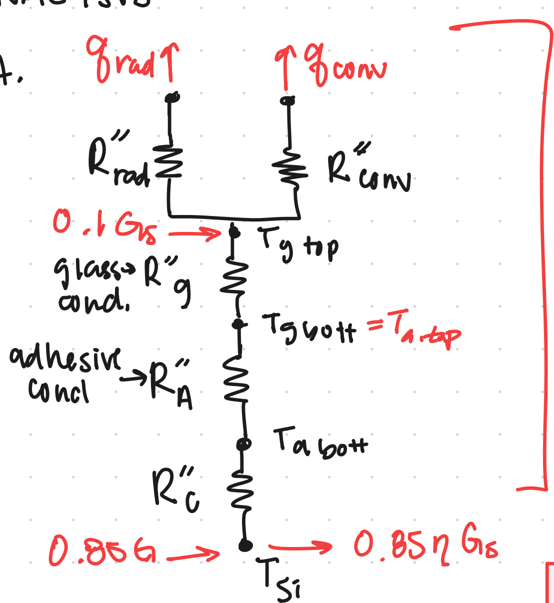
SCHEMATIC:





ANALYSIS:

A.



$\boxed{R''_c = 10^{-4}\ \frac{m^2K}{W}}$

$R''_{conv} = \frac{1}{h}$ → $h = 10\ W/m^2K$ → $\boxed{0.1\ m^2K/W = R''_{conv}}$ A

$R''_{rad} = \frac{1}{h_{rad}}$ $h_{rad} = 5\ W/m^2K$ → $\boxed{0.2\ m^2K/W = R''_{rad}}$

$R''_g = \frac{L_g}{k_g}$ $k_g = 1.5\ W/mK$ $L_g = .003\ m$ → $\boxed{.002\ m^2K/W = R''_g}$

$R''_a = \frac{L_a}{k_a}$ $k_a = 1\ W/mK$ $L_a = .0001\ m$ → $\boxed{.0001\ m^2K/W = R''_a}$

(B) ENERGY BALANCE OF SILICON LAYER:

$$0.85\,G_s - 0.85\,G_s\eta = \frac{T_{Si} - T_{g\,top}}{R''_c + R''_g + R''_A} = \frac{T_{Si} - T_{gtop}}{}$$

$$0.85\,G(1-a+bT_{Si}) = \frac{T_{Si} - T_{gtop}}{R''_c + R''_A + R''_g}$$

$$680(1-0.553+0.001\,T_{Si}) = \frac{T_{Si} - T_{gtop}}{.0001 + .002 + .0001}$$

$$1.496(0.447 + .001X) = X - Y \quad (1)$$

ENERGY BALANCE OF TOP SURFACE OF GLASS:

$$\cancel{kW}\,(0.85\,G_s(1-\underbrace{\eta}_{a-bT_{Si}}) + 0.1\,G_s) = [h(T_{g\,top} - T_\infty) - h_{rad}(T_{sur} - T_{gtop})]\,\cancel{kW}$$

$$680(1-a+bT_{Si}) + 80 = 10(T_{gtop} - 300) - 5(300 - T_{gtop}) \quad (2)$$

$$680 - 0.553(680) + .68\underbrace{T_{Si}}_{X} + 80 = 10(\underbrace{T_{gtop}}_{Y} - 300) - 5(300 - \underbrace{T_{gtop}}_{Y})$$

SOE SOLVER:

$\boxed{T_{Si} = 342.296\,K}$ $\boxed{T_{gtop} = 341.115\,K}$ (B)

(C) $\eta = 0.553 - 0.001(342.296) = 0.2107$

$\boxed{21.07\%}$

$P = 0.85\,G\,\eta$

$$\boxed{P = 143.278\,\frac{W}{m^2}}$$

② KNOWN: $D = 10\,mm$ $\quad T_w = 200°C$ $\quad L_0$ $\quad T_\infty = 20°C$

$k = 80\,W/mK$ $\quad L_{ins} = 200\,mm$ $\quad T_{max} = 100°C$ $\quad h = 20\,W/m^2K$

FIND:

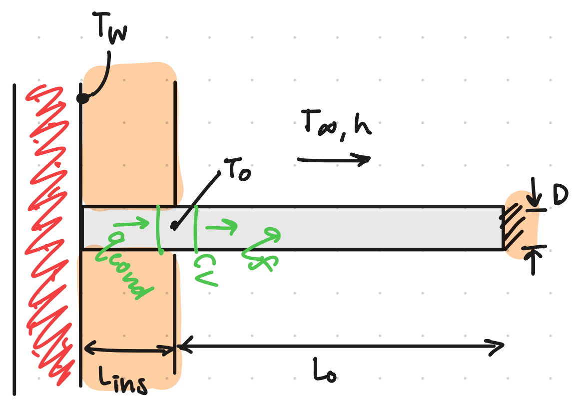
A. derive an expression for T_0

B. if $L_0 = 200\,mm$, calc T_0, does the length meet the operating limit?

C. calc. min L_0 (s.t. $T_0 <$ operating lim)

ASSUMPTIONS: 1D, SS, adiabatic tip

SCHEMATIC:



ANALYSIS:

ADIABATIC: $\dfrac{\theta}{\theta_b} = \dfrac{\cosh m(L-x)}{\cosh(mL)}$ TABLE 3.4

$m = \sqrt{\dfrac{hP}{kA_c}}$ $\qquad M = \sqrt{hPkA_c}\,\theta_b$

$A_c = \pi r^2$ $\qquad P = 2\pi r$

$P = 2\pi r = \pi D = .01\pi$

$A_c = \pi r^2 = \pi \dfrac{D^2}{4} = .0025\pi$

$m = \sqrt{\dfrac{hP}{kA_c}} = \sqrt{\dfrac{20(.01\cancel{\pi})}{80\,\cancel{\pi}\,.0025}} = 1$

$M = \sqrt{20(.01\pi)(80)(.0025\pi)}\;\theta_b$ $\qquad q_f = M\tanh mL$

ENERGY BALANCE @ INS. $\qquad \theta_b = T_0 - T_\infty$ $\qquad q_f = q_{cond}$

$q_f = q_{cond}$

$M \tanh mL_0 = \dfrac{kA_c}{L_{ins}}(T_w - T_0)$

$\underbrace{M \tanh mL_0}_{\sqrt{hPkA_c}\,(T_0 - T_\infty)\tanh mL_0} = \dfrac{kA_c(T_W - T_0)}{L_{ins}}$

$\underbrace{\sqrt{hPkA_c}\,\tanh mL_0}_{B}(T_0 - T_\infty) = \underbrace{\dfrac{kA_c}{L_{ins}}}_{x}(T_W - T_0)$

$BT_0 - BT_\infty = xT_W - xT_0$

$(B + x)T_0 = xT_W + BT_\infty$ $\qquad T_0 = \dfrac{xT_W + BT_\infty}{B + x}$

$X = \frac{KA_c}{L_{ins}}$ $B = \sqrt{hPKA_c}\tanh mL_o$ $\frac{1}{B} = R_{fin}$

$$\boxed{T_o = \frac{\frac{KA_c}{L_{ins}} T_W + \sqrt{hPKA_c}\tanh(mL_o)T_\infty}{\frac{KA_c}{L_{ins}} + \sqrt{hPKA_c}\tanh mL_o}}$$

$$\frac{\frac{1}{R_{ins}} T_W + \frac{1}{R_{fin}} T_\infty}{\quad}$$

B. $D = 10\,mm$ $T_W = 200°C$ L_o $T_\infty = 20°C$ $P = 2\pi r = \pi D = .01\pi$

$K = 80\,W/mK$ $L_{ins} = 200mm$ $T_{max} = 100°C$ $h = 20\,W/m^2K$ $A_c = \pi r^2 = .0025\pi$

$P = 2\pi r = \pi D = .01\pi$

$A_c = \pi r^2 = \pi \frac{D^2}{4} = .0025\pi$

$\boxed{T_o = 81.47°C}$ USED SYMBOLAB

Yes, this does stay under the operating condition

C. Use T_o and solve for lowest L_o:

$$T_o = \frac{\frac{KA_c}{L_{ins}} T_W + \sqrt{hPKA_c}\tanh(mL_o)T_\infty}{\frac{KA_c}{L_{ins}} + \sqrt{hPKA_c}\tanh mL_o}$$

mm & E

$$100°C > \frac{\frac{80(.0025\pi)}{.2}(200) + \sqrt{20(.01\pi)80(.0025\pi)}\tanh(10L_o)20°C}{\frac{80(.0025\pi)}{.2} + \sqrt{20(.01\pi)80(.0025\pi)}\tanh(10L_o)}$$

$\boxed{L_o = .073\,m}$

(3) KNOWN: $k = 400\ W/mK$ $s = 10mm$ $h = 100\ W/m^2K$ $\eta_f = 0.6$

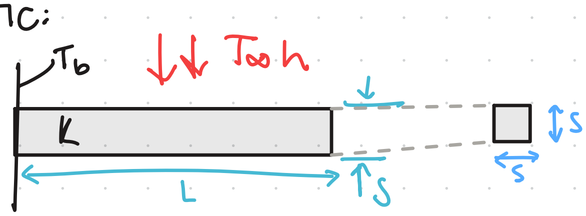
FIND:

A. calc L

B. calc therm resist. of R_f & ε_f

ASSUMPTIONS: 1D, SS, convective tip

SCHEMATIC:



ANALYSIS:

$(.01)^2 = .0001$

$q_f = \eta_f h A_f \theta_b$

$q_{max} = h A_f \theta_b$

A. $\eta_f = \dfrac{\tanh mL_c}{mL_c}$

$L_c = .151\ m$

$M = \sqrt{hPkA_c}\,\theta_b$

$m = \sqrt{\dfrac{hP}{kA_c}} = \sqrt{\dfrac{100(.04)}{400(.0001)}} =$

table 3.5 $L_c = L + t/2$

$.151 = L + .01/2$

$\boxed{L = .146\,m}$

$k = 400\ W/mK$ $s = 10mm$ $h = 100\ W/m^2K$

B. $\eta_f = 0.6$

$$q_f = M\left(\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}\right)$$

$$q_f = \theta_b\sqrt{hPkA_c}\left[\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}\right]$$

$$R_f = \frac{\theta_b}{q_f} = \frac{\cancel{\theta_b}}{\sqrt{hPkA_c}\,\cancel{\theta_b}\,(\sim)} = \frac{1}{\sqrt{hPkA_c}\,(\sim)}$$

$$R_f = \frac{1}{\underbrace{\sqrt{(100)\,.04\,(400)(.0001)}}_{0.4}\;\dfrac{\sinh 10\cdot(.146) + \frac{100}{10\,(400)}\cosh 10\,(.146)}{\cosh 10\,(.146) + \underbrace{\frac{100}{10\,(400)}}_{.025}\sinh 10\cdot(.146)}}$$

$$\boxed{R_f = 2.77\ \frac{K}{W}}$$

$$\varepsilon_f = \frac{q_f}{hA_c\theta_b} = \frac{\sqrt{hPkA_c}\,\cancel{\theta_b}}{hA_c\,\cancel{\theta_b}}\left[\sim\right] = \boxed{36.1 = \varepsilon_f}$$